LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – NOVEMBER 2007

MT 3803/MT 3800 - TOPOLOGY

AB 24

Date : 24/10/2007 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

Answer all questions. All questions carry equal marks.

01.(a)(i) Let X be a metric space with metric d. Show that d_1 defined by
$d_1(x,y) = \dfrac{d(x,y)}{1+d(x,y)}$ is also a metric on X.

(OR)

(ii) Let X be a metric space. Show that any union of open sets in X is open and any finite intersection of open sets in X is open. (5)

(b)(i) Let X be a metric space, and let Y be a subspace of X. Prove that Y is complete iff Y is closed.

(ii) State and prove Cantor's Intersection Theorem.

(iii) State and prove Baire's Theorem. (6+ 5 + 4)

(OR)

(iv) Let X be a metric space. Prove that a subset G of X is open $\Leftrightarrow$ it is a union of open spheres.

(v) Let f: X $\rightarrow$ Y be a mapping of one topological space into another.
Show that f is continuous $\Leftrightarrow$ $f^{-1}(F)$ is closed in X whenever F is closed is Y $\Leftrightarrow$ $f(\overline{A}) \subseteq \overline{f(A)}$ $\forall$ A $\subseteq$ X. (6 + 9)

02.(a)(i) Prove that every separable metric space is second countable.

(OR)

(ii) If f and g are continuous real or complex functions defined on a topological space X, then show that f + g and αf are also continuous. (5)

(b)(iii)Show that any continuous image of a compact space is compact.

(iv)Prove that any closed subspace of a compact space is compact.

(v) Give an example to show that a compact subspace of a compact sphere need not be closed. (6 + 6 + 3)

(OR)

(i) Let C(X, R) be the set of all bounded continuous real function defined on a topological space X. Show that (i) C(X, R) is a real Banach space with respect to positive condition and multiplication and the norm defined by

$\|f\| = \sup|f(x)|$;   (ii)If multiplication is defined pointwise C(X, R) is a commutative real algebra with identity in which $\|fg\| \leq \|f\| \|g\|$ and $\|f\| = 1$.
(10 + 5)

03.(a) (i) Prove that the product of any non–empty class of compact spaces is compact.

(OR)

(ii) Show that every sequentially compact metric space is compact.   (5)

(b) (i) Show that a metric space is sequentially compact $\Leftrightarrow$ it has the Bolzero Weierstrass property.

(ii) Prove that every compact metric space has the Bolzano Weierstrass property.

(OR)   (10 + 5)

(iii) Prove that a metric space is compact $\Leftrightarrow$ it is complete and totally bounded.

(iv) Let X be a compact metric space. If a closed subspace of C(X, R) or C(X, C) is compact, show that it is bounded and equicontinuous.   (7 + 8)

04.(a)(i) Prove that every compact Hausdorff space is normal.

(OR)

(ii) Let X be a T_1–space. Show that X is normal $\Leftrightarrow$ each neighbourhood of a closed set F contains the closure of some neighbourhood of F.   (5)

(b)(iii)Show that every subspace of a Hausdorff space is also Hausdorff.

(iv) Prove that every compact subspace of a Hausdorff space is closed.

(v) Show that a one–to–one continuous mapping of a compact space onto a Hausdorff space is a homemorphism.   (6 + 4 + 5)

(OR)

(i) State and prove the Uryshon Imbedding Theorem.

05.(a) (i) Prove that any continuous image of a connected space to connected.

(OR)

(ii) Let X be a topological space and A be a connected subspace of X.   (5)
If B is a subspace of X such that A $\subseteq B \subseteq \overline{A}$, show that B is connected.

(b) (i) Show that a subspace of a real line R is connected $\Leftrightarrow$ it is an interval.

(ii) Prove that the product of any non–empty class of connected spaces is connected.   (9 + 6)

(OR)

(iii) State and prove the Weierstrass Approximation Theorem.   (15)

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